BLSC seminar 2015. 12. 17

Brain Science Inspired Life Support Research Center

一次視覚野自己組織化における未解決問題に対する試論

脳科学ライフサポート研究センター
総合コミュニケーション科学推進室
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Toward Unified Self-Organization Theory of Mammalian Primary Visual Cortex

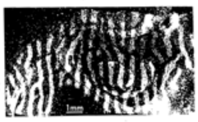
- LGN neurons' response properties (Nonlagged vs. lagged cells, triadic synapses)
- Synaptic rewiring mechanisms (LTP, LTD, synaptic competition, etc.)
- Molecular mechanisms of synaptic plasticity (Ca^{2+}, AMPAR, NMDAR, VDCC, kainese vs. phosphatase, neurotrophic factors, cytoskeletal proteins)
- Visual feature extraction and representation (Orientation, direction of motion, spatial frequency, ocular dominance, binocular disparity and retinotopy)
- Emergence of simple/complex cells (F1/F0, contrast invariance)
- Map representation vs. salt-and-pepper representation (cats , ferrets, old-world monkeys, new-world monkeys, tree shrew and rodents)
- Sensitivity period (critical period)

中世ルネッサンスのフィレンツェにおいては、専門領域を越えてお互いに影響を与え合い、技術や文化がめざましく発展を遂げた。著者は、その繁栄を「メディチ・エフェクト」と名づけ、イノベーションにおいても異文化・異分野が交差するところにおいてこそ、斬新な発明・アイデアが生まれる、と説く。このメディチ・エフェクトを意図的につくりだす方法を、古今東西の発明家、起業家、芸術家、建築家、科学者の実例を紹介しながら説明する。

本日のお話

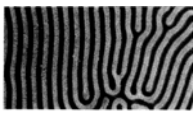
物理学と神経科学の交差点

眼優位マップ



マカクザル 一次視覚野
Hubel, Wiesel, LeVay (1977)

アナロジー



磁性薄膜のドメイン構造

自然界にある縞模様

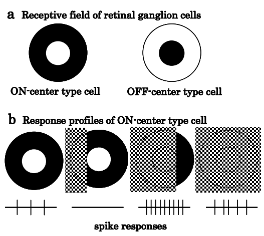
シマウマの縞模様

ロイヤルプレコ

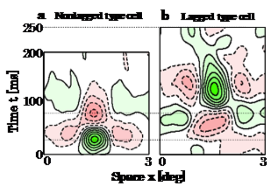
イシダイ

外側膝状体(LGN)ニューロンの反応特性

受容野の空間パターン

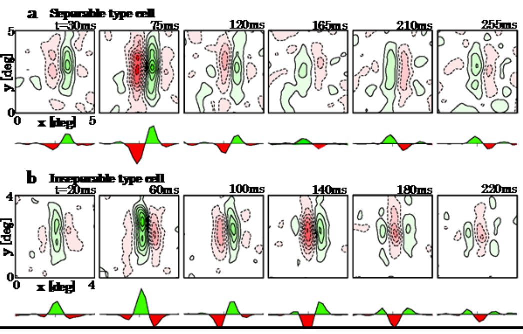


受容野の時空間パターン



視覚野4層ニューロンの反応特性

単純型の時空間受容野



受容野の自発的対称性の破れ

	同心円状の受容野 （オン中型、オフ中心型）	Ganglion cells in retina, Relay cells in LGN
	↓ !	
Klein Bottle	方位選択的な受容野 （方位、位相、運動方向）	Simple cells in V1
	↓ ?	
Möbius Band	方位選択的な受容野 （方位、運動方向）	Complex cells in V1

シナプス可塑性の数理モデル（1）

Membrane potential and firing rate of Layer 2/3 neuron *i*

$$\frac{d}{dt}\varsigma_j^{\mathrm{L2/3},\nu}(t) = -\frac{1}{\tau_\nu}\varsigma_j^{\mathrm{L2/3},\nu}(t) + (\text{Input from L4 neurons}) + (\text{Input from other L2/3 neurons})$$

$$\eta_j^{\mathrm{L2/3}}(t) = F_\vartheta\left(\varsigma_j^{\mathrm{L2/3},fast}(t) + r\varsigma_j^{\mathrm{L2/3},slow}(t)\right)$$

Membrane potential and firing rate of Layer 4 neuron *i*

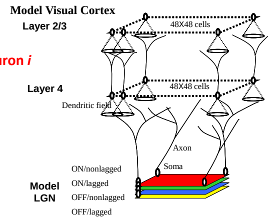
$$\frac{d}{dt}\varsigma_i^{\mathrm{L4}}(t) = -\frac{1}{\tau_{fast}}\varsigma_i^{\mathrm{L4}}(t) + (\text{Input from LGN neurons}) + (\text{Input from other L4 neurons})$$

$$\eta_i^{L4}(t) = F_\vartheta\left(\varsigma_i^{\mathrm{L4}}(t)\right)$$

Gated Hebbian coincidence + synaptic constraint

$$h_{i;k,\mu}^{LGN-L4} = \left\langle \xi_i^{L4}\varsigma_i^{L4}(t)\eta_{k,\mu}^{LGN}(t)\right\rangle - c\sigma_{i;k,\mu}$$

$$h_{j;i}^{L4-L2/3} = \left\langle \xi_j^{L2/3}\varsigma_j^{L2/3}(t)\eta_i^{L4}(t)\right\rangle - c\sigma_{j;i}$$



シナプス可塑性の数理モデル（2）

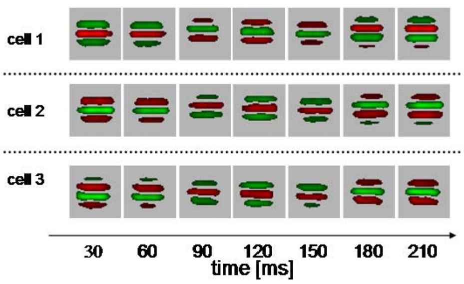
Probability of synaptic rewiring

$$P_{LGN-L4}\left(\sigma_{i;k,\mu} \to \sigma_{i;k,\mu}+1,\ \sigma_{i;k',\mu'} \to \sigma_{i;k',\mu'}-1\right) = \frac{1}{1+e^{-\beta\left(h_{i;k,\mu}^{LGN-L4} - h_{i;k',\mu'}^{LGN-L4}\right)}}$$

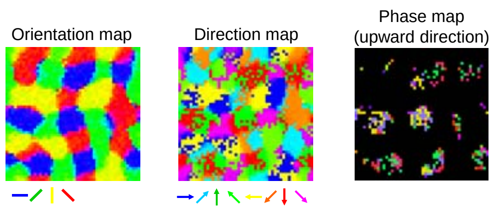
$$P_{L4-L2/3}\left(\sigma_{j;i} \to \sigma_{j;i}+1,\ \sigma_{j;i'} \to \sigma_{j;i'}-1\right) = \frac{1}{1+e^{-\beta\left(h_{j;i}^{L4-L2/3} - h_{j;i'}^{L4-L2/3}\right)}}$$

2-synapse fillip ➡ synaptic number conservation

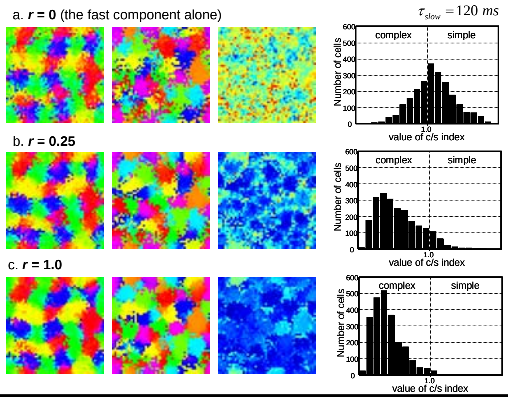
RFs of typical L4 cells



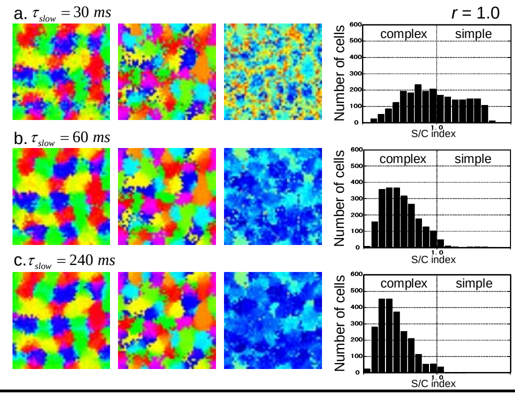
L4の特徴マップ



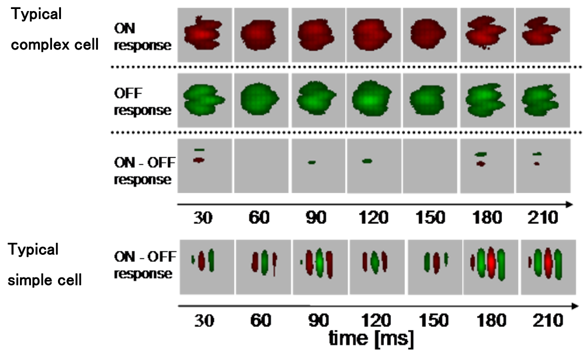
Contribution of the slow component



Slow decay constant dependence



RFs of L2/3 cells



複雑型細胞形成の主要因

Fast decay time membrane potential is assumed

(AMPAR & NR2A-containing NMDAR)

# of simple cells ># of complex cells

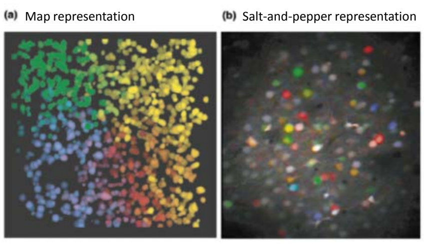
Fast & slow decay time membrane potentials are assumed

(AMPAR, NR2A-containing NMDAR & NR2B-containing NMDAR)

# of complex cells > # of simple cells

Formation of complex cells requires NR2B containing NMDAR.

なぜ、ネコとネズミの方位マップは質的に異なる？



Single-cell resolution orientation maps from (a) a pinwheel in cat visual cortex and (b) rat visual cortex obtained with in vivo two-photon calcium imaging. One side is 300 mm. Cells are colored according to their preferred orientation. From Ohki & Reid, 2007

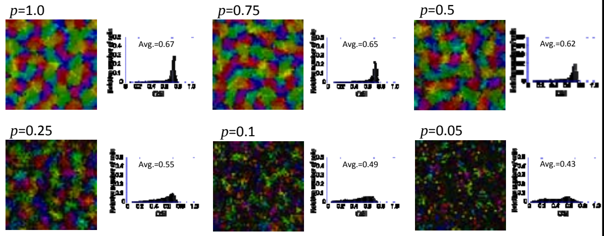
「興奮性疎結合」のモデル

$$\zeta_i^{LT} = \sum_{i' \neq i} V_{i;i'}^{CX} \eta_{i'}^{CX}$$

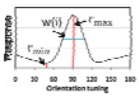
$$V_{i;i'}^{CX} = \frac{\sigma_{i,i'}}{p} \cdot \frac{1}{2\pi\lambda_{ex}^2} \exp\left(-\frac{d_{i;i'}^2}{2\lambda_{ex}^2}\right) - \frac{1}{2\pi\lambda_{inh}^2} \exp\left(-\frac{d_{i;i'}^2}{2\lambda_{inh}^2}\right)$$

$\sigma_{i,i'} = 1$ or 0, $\langle \sigma_{i,i'} \rangle = p$: excitatory connection probability

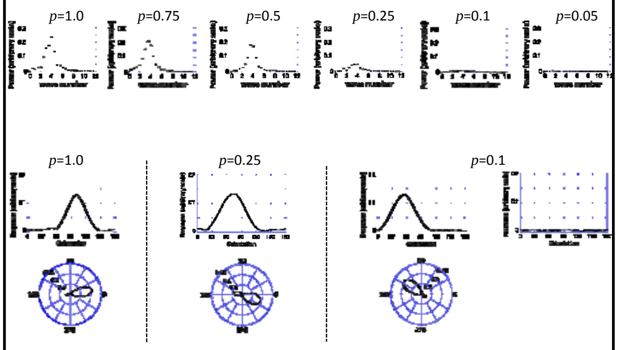
Orientation representations in L4



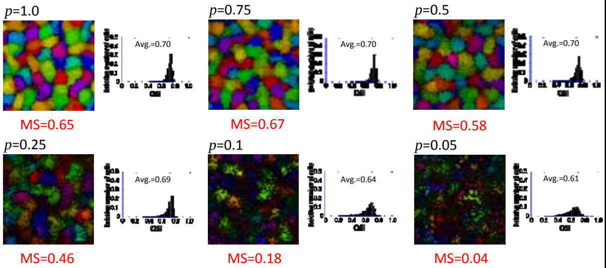
$$OSI = \frac{1}{N}\sum_i \left(1-\frac{r_{min}(i)}{r_{max}(i)}\right)\left(1-\frac{w(i)}{180}\right)$$



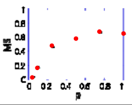
Power spectra & tuning curves



Orientation representation in L2/3



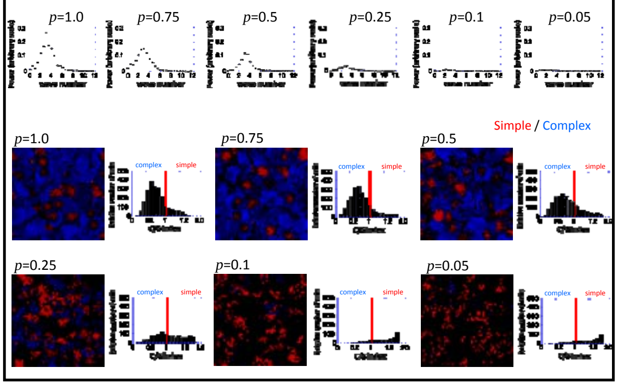
MS vs. P



MS: Map similarity between Lay4 and layer 2/3

$$MS = \frac{1}{N}\sum_i \cos\left[2(\theta_i^{L4} - \theta_i^{L2/3})\right]$$

Power spectra & s/c distributions



「興奮性疎結合」のモデルは尤もらしいか？

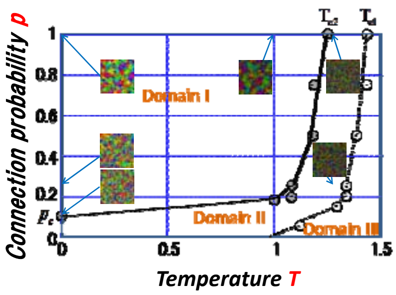
For p>0.1, Orientations are represented in a regular map
Complex cells are rich
Maps in L2/3 are juxtaposed with maps in L4

For p<0.1, Orientation representation was salt-and-pepper-like,
Simple cells are rich
Maps differ between L4 and L2/3

モデルからの予言

Cats → p>0.1, and rodents → p<0.1

T-p space



Domain I
- Oriented RFs
- Map representation

Domain II
- Oriented RFs
- Random representation

Domain III
- Unoriented RFs

Derivation of Spin Hamiltonian

$$G=\frac{1}{2}\sum_i\sum_{k,\mu}\sum_{\substack{k',\mu'\\ \neq k,\mu}}\sigma_{i,k',\mu'}\sigma_{i,k,\mu}\left\langle \xi_i^{CX}\eta_{k',\mu'}^{LGN}\eta_{k,\mu}^{LGN}\right\rangle+\frac{1}{2}\sum_i\sum_{i'\neq i}V_{i,i'}\left\langle \eta_i^{CX}\eta_{i'}^{CX}\right\rangle$$

Gate function: $\xi_i^{CX}=\begin{cases}1 & \text{for } \varsigma_i^{CX}>0\\ 0 & \text{for } \varsigma_i^{CX}<0\end{cases}$

$$\frac{\partial G}{\partial\sigma_{i,k,\mu}}=\left\langle \sum_{\substack{k',\mu'\\ \neq k,\mu}}\sigma_{i,k',\mu'}\xi_i^{CX}\eta_{k',\mu'}^{LGN}\eta_{k,\mu}^{LGN}+\xi_i^{CX}\eta_{k,\mu}^{LGN}\sum_{i'\neq i}V_{i,i'}\eta_{i'}^{CX}\right\rangle$$

$$=\left\langle \xi_i^{CX}\left(\sum_{\substack{k',\mu'\\ \neq k,\mu}}\sigma_{i,k',\mu'}\eta_{k',\mu'}^{LGN}+\sum_{i'\neq i}V_{i,i'}\eta_{i'}^{CX}\right)\eta_{k,\mu}^{LGN}\right\rangle$$

$=\left\langle \xi_i^{CX}\varsigma_i^{CX}\eta_{k,\mu}^{LGN}\right\rangle$ Gated Hebbian Coincidence

Derivation of Spin Hamiltonian

$$G=G^{(0)}+G^{(1)}$$

$$G^{(1)}=\frac{1}{2}\sum_i\sum_{i'\neq i}V_{i,i'}\left\langle \eta_i^{CX}\eta_{i'}^{CX}\right\rangle$$

$$\eta_i^{CX}=\bar{\eta}\frac{1+\cos 2\left(\theta^{stim}-\theta_i\right)}{2}$$

$$G^{(1)}=\frac{\bar{\eta}^2}{2}\sum_i\sum_{i'\neq i}V_{i,i'}\frac{1}{\pi}\int_0^{\pi}\frac{1+\cos 2\left(\theta^{stim}-\theta_i\right)}{2}\frac{1+\cos 2\left(\theta^{stim}-\theta_{i'}\right)}{2}d\theta^{stim}$$

$$=\frac{\bar{\eta}^2}{16}\sum_i\sum_{i'\neq i}V_{i,i'}\cos 2\left(\theta_i-\theta_{i'}\right)+\frac{\bar{\eta}^2}{8}$$

$$H=-\frac{1}{2}\sum_i\sum_{i'\neq i}V_{i,i'}\cos 2\left(\theta_i-\theta_{i'}\right)$$

$$=-\frac{1}{2}\sum_i\sum_{i'\neq i}V_{i,i'}\vec{S}_i\cdot\vec{S}_{i'}$$

Afferent input induced preferred orientation:

$$\vec{S}_j=\left(S_{j,x},\ S_{j,y}\right)=\left(\cos 2\theta_j,\ \sin 2\theta_j\right)$$

スピン系の物理による Orientation representationの理解

Hamiltonian (energy function, cost function):

$$H=-\frac{1}{2}\sum_{\substack{j,j'\\ j\neq j}}\left(V_{j,j'}^{ex}-V_{j,j'}^{inh}\right)\vec{S}_j\cdot\vec{S}_{j'}-\sum_j\vec{h}\cdot\vec{S}_j$$

Intracortical excitatory and inhibitory connections:

$$V_{j,j'}^{ex}=\frac{1}{2\pi\Lambda_{ex}^2}e^{-\frac{\left(\vec{x}_j-\vec{x}_{j'}\right)^2}{2\lambda_{ex}^2}}\qquad \lambda_{ex}=\Lambda_{ex}\Delta x$$

$$V_{j,j'}^{inh}=\frac{\kappa}{2\pi\Lambda_{inh}^2}e^{-\frac{\left(\vec{x}_j-\vec{x}_{j'}\right)^2}{2\lambda_{inh}^2}}\qquad \lambda_{inh}=\Lambda_{inh}\Delta x$$

Thermodynamics of orientation representations

Partition function:

$$Z=\underset{\{\vec{S}_j\}}{Tr}\,e^{-\beta H}$$

$$=\underset{\{\vec{S}_j\}}{Tr}\,e^{\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}V_{j,j'}\vec{S}_j\cdot\vec{S}_{j'}+\beta\sum_j\vec{h}\cdot\vec{S}_j}$$

Method of auxiliary variables:

$$Z=\int Dm_j\underset{\{\vec{S}_j\}}{Tr}\,e^{-\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\vec{m}_{j'}+\beta\sum_j\left(\vec{m}_j+\vec{h}\right)\cdot\vec{S}_j}$$

$$=\int Dm_j e^{-\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\vec{m}_{j'}}\underset{\{\vec{S}_j\}}{Tr}\,e^{\beta\sum_j\left(\vec{m}_j+\vec{h}\right)\cdot\vec{S}_j}$$

$$=\int Dm_j e^{-\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\vec{m}_{j'}}\prod_j\int_0^{\pi}\frac{d\theta_j}{\pi}e^{\beta\left[\left(m_{j,x}+h_x\right)\cos 2\theta_j+\left(m_{j,y}+h_y\right)\sin 2\theta_j\right]}$$

$$=\int Dm_j e^{-\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\vec{m}_{j'}}\prod_j I_0\left(\beta\sqrt{\left(\vec{m}_j+\vec{h}\right)^2}\right)$$

$$=\int Dm_j e^{-\frac{\beta}{2}\sum_{\substack{j,j'\\ j\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\vec{m}_{j'}+\sum_j\ln I_0\left(\beta\sqrt{\left(\vec{m}_j+\vec{h}\right)^2}\right)}$$

Derivation of mean field free energy

$$-\frac{\beta}{2}\sum_{\substack{j,j'\\ j'\neq j}}(V)^{-1}{}_{j,j'}\vec{m}_j\cdot\vec{m}_{j'}$$

$$=-\frac{\beta}{2}\frac{1}{(2\pi)^2}\int d\vec{k}\,\frac{1}{\tilde{V}\left(\vec{k}\right)}\tilde{\vec{m}}\left(-\vec{k}\right)\cdot\tilde{\vec{m}}\left(\vec{k}\right)$$

$$\simeq-\frac{\beta}{2}\frac{1}{(2\pi)^2}\int d\vec{k}\,\tilde{\vec{m}}\left(-\vec{k}\right)\cdot\left(\frac{1}{\tilde{V}\left(k_0\right)}+\frac{\left|\tilde{V}''\left(k_0\right)\right|}{8\tilde{V}\left(k_0\right)^3\vec{k}_0^2}\left(\vec{k}^2-\vec{k}_0^2\right)^2\right)\tilde{\vec{m}}\left(\vec{k}\right)$$

$$\simeq-\frac{\beta}{2}\frac{1}{\tilde{V}\left(k_0\right)}\int d\vec{x}\left[\left|\vec{m}\left(\vec{x}\right)\right|^2+\vec{m}\left(\vec{x}\right)\cdot\frac{\left|\tilde{V}''\left(k_0\right)\right|}{8k_0^2\tilde{V}\left(k_0\right)^2}\left(\vec{\nabla}^2+k_0^2\right)^2\vec{m}\left(\vec{x}\right)\right]$$

k_0 **is determined by a solution to the equation** $\frac{d}{dk}\tilde{V}\left(k\right)=0$

Free energy of continuous mean field

$$F=\frac{1}{2}\int d\vec{x}\left[\tilde{V}\left(k_0\right)\left|\vec{m}\left(\vec{x}\right)\right|^2+\vec{m}\left(\vec{x}\right)\cdot\frac{\left|\tilde{V}''\left(k_0\right)\right|}{8k_0^2\tilde{V}\left(k_0\right)^3}\left(\vec{\nabla}^2+k_0^2\right)^2\vec{m}\left(\vec{x}\right)-\frac{2}{\beta}\ln I_0\left(\beta\sqrt{\left(\vec{m}\left(\vec{x}\right)+\vec{h}\right)^2}\right)\right]$$

Critical temperature for the emergence of nonzero mean field

Relaxation dynamics of the mean field near equilibrium

$$\frac{d\vec{m}\left(\vec{x}\right)}{dt}=-\frac{\partial F}{\partial\vec{m}\left(\vec{x}\right)}$$

2D-Swift-Hohenberg equation

$$\frac{d\vec{m}\left(\vec{x}\right)}{dt}=\left(\frac{\beta}{2}-\frac{1}{\tilde{V}\left(k_0\right)}\right)\vec{m}\left(\vec{x}\right)-\frac{\left|\tilde{V}''\left(k_0\right)\right|}{8k_0^2\tilde{V}\left(k_0\right)^3}\left(\vec{\nabla}^2+k_0^2\right)^2\vec{m}\left(\vec{x}\right)-\frac{\beta^3}{16}\left|\vec{m}\left(\vec{x}\right)\right|^2\vec{m}\left(\vec{x}\right)$$

Critical temperature *Tc* is determined by $\frac{\beta_c}{2}-\frac{1}{\tilde{V}\left(k_0\right)}=0$

$$T_c=\frac{\tilde{V}\left(k_0\right)}{2}$$

Sparse excitatory connections

$$H = -\frac{1}{2p}\sum_{\substack{j,j'\\ j\neq j}} V^{ex}_{j,j'}\sigma_{j,j'}\vec{S}_j\cdot\vec{S}_{j'} + \frac{1}{2}\sum_{\substack{j,j'\\ j\neq j}} V^{inh}_{j,j'}\vec{S}_j\cdot\vec{S}_{j'} - \sum_j \vec{h}\cdot\vec{S}_j$$

$\sigma_{j,j'}$ represents a binary stochastic variable taking 1 or 0, which indicates a presence or absence of a synaptic connection between sites *j* and *j'*.

The mean value of $\sigma_{j,j'}$ is give by *p*.

$$F = -T\left\langle \ln Z\right\rangle_{[\sigma]} = -\frac{1}{\beta}\left\langle \ln\left(\underset{[\vec{S}]}{Tr}\, e^{-\beta H}\right)\right\rangle_{[\sigma]}$$

$$= -\frac{1}{\beta}\left\langle \lim_{n\to 0}\frac{Z^n - 1}{n}\right\rangle_{[\sigma]}$$ ← Replica trick

$$= -\frac{1}{\beta}\lim_{n\to 0}\frac{\left\langle \underset{[\vec{S}]}{Tr}\, e^{-\beta\sum_\alpha H\left(\left[\vec{S}^\alpha\right]\right)}\right\rangle_{[\sigma]} - 1}{n}$$

$$\left\langle Z^n\right\rangle = \underset{[\vec{S}^\alpha]}{Tr}\left(e^{-\frac{\beta}{2}\sum_{(j,j')} V^{inh}_{j,j'}\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'} + \beta\sum_j \vec{h}\cdot\sum_\alpha \vec{S}^\alpha_j}\left\langle e^{\frac{\beta}{2p}\sum_{(j,j')} V^{ex}_{j,j'}\sigma_{j,j'}\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'}}\right\rangle_{[\sigma]}\right)$$

$$= \underset{[\vec{S}^\alpha]}{Tr}\left(e^{-\frac{\beta}{2}\sum_{(j,j')} V^{inh}_{j,j'}\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'} + \beta\sum_j \vec{h}\cdot\sum_\alpha \vec{S}^\alpha_j + \sum_{(j,j')}\ln\left(p e^{\frac{\beta}{2p}V^{ex}_{j,j'}\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'}} + 1 - p\right)}\right)$$

$$\approx \underset{[\vec{S}^\alpha]}{Tr}\left(e^{\frac{\beta}{2}\sum_{(j,j')}\left(V^{ex}_{j,j'} - V^{inh}_{j,j'}\right)\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'} + \beta\sum_j \vec{h}\cdot\sum_\alpha \vec{S}^\alpha_j + \frac{\beta^2(1-p)}{8p}\sum_{(j,j')}\left(V^{ex}_{j,j'}\right)^2\left(\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'}\right)^2}\right)$$

$$= \int D\vec{q}^{\,\alpha\alpha'}_j\, e^{-\frac{\beta\gamma}{4}\sum_{\alpha,\alpha'}\sum_{(j,j')}\left[\left(V^{ex2}\right)^{-1}\right]_{j,j'}\vec{q}^{\,\alpha\alpha'}_j\cdot A^{-1}\vec{q}^{\,\alpha\alpha'}_{j'}}$$

$$\times \underset{[\vec{S}^\alpha]}{Tr}\left(e^{\frac{\beta}{2}\sum_{(j,j')}\left(V^{ex}_{j,j'} - V^{inh}_{j,j'}\right)\sum_\alpha \vec{S}^\alpha_j\cdot\vec{S}^\alpha_{j'} + \beta\sum_j \vec{h}\cdot\sum_\alpha \vec{S}^\alpha_j + \frac{\beta\gamma}{2}\sum_{\alpha,\alpha'}\sum_j \vec{q}^{\,\alpha\alpha'}_j\cdot\vec{u}^{\,\alpha\alpha'}_j}\right)$$

$$\vec{u}^{\,\alpha\alpha'}_j = \left(S^\alpha_{j,x}S^{\alpha'}_{j,x} \quad S^\alpha_{j,y}S^{\alpha'}_{j,y} \quad S^\alpha_{j,x}S^{\alpha'}_{j,y}\right)$$

$$A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$\vec{q}^{\,\alpha\alpha'}_j = \left(q^{\alpha\alpha'}_{j,1} \quad q^{\alpha\alpha'}_{j,2} \quad q^{\alpha\alpha'}_{j,3}\right)$$

$$\gamma = \frac{\beta(1-p)}{2p}$$

We will search for replica symmetric solutions.

$$q^{\alpha\alpha'}_{j,1} = q^{\alpha\alpha'}_{j,2} = q^{INT}_j \quad (\alpha' \neq \alpha),\quad q^{\alpha\alpha}_{j,1} = q^{\alpha\alpha}_{j,2} = q^{SELF}_j,\quad q^{\alpha\alpha'}_{j,3} = 0$$

$$\left\langle \ln Z\right\rangle = \lim_{n\to 0}\frac{1}{n}\left(\left\langle Z^n\right\rangle - 1\right)$$

$$= \lim_{n\to 0}\frac{1}{n}\left[e^{-\frac{\beta\gamma}{2}n(n-1)\sum_{(j,j')}\left[\left(V^{ex2}\right)^{-1}\right]_{j,j'} q^{INT}_j q^{INT}_{j'} - \frac{\beta\gamma}{2}n\sum_{(j,j')}\left[\left(V^{ex2}\right)^{-1}\right]_{j,j'} q^{SELF}_j q^{SELF}_{j'} - \frac{\beta}{2}n\sum_{(j,j')}\left[\left(V^{ex} - V^{inh}\right)^{-1}\right]_{j,j'}\vec{m}_j\cdot\vec{m}_{j'}}\right.$$

$$\left.\times e^{n\frac{\beta\gamma}{2}\left(\sum_j q^{SELF}_j - \sum_j q^{INT}_j\right)}\prod_j\int\frac{1}{2\pi}d\vec{z}_j e^{-\frac{1}{2}\left(\vec{z}_j\right)^2}\left(\underset{[\vec{S}_j]}{Tr}\left(e^{\beta\left(\vec{m}^\alpha_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j\right)\cdot\vec{S}_j}\right)\right)^n - 1\right]$$

$$= -\frac{\beta}{2}\sum_{(j,j')}\left[\left(V^{ex} - V^{inh}\right)^{-1}\right]_{j,j'}\vec{m}_j\cdot\vec{m}_{j'}$$

$$-\frac{\beta\gamma}{2}\sum_{(j,j')}\left[\left(V^{ex2}\right)^{-1}\right]_{j,j'}\left(q^{SELF}_j - \frac{\rho}{2}\right)\left(q^{SELF}_{j'} - \frac{\rho}{2}\right)$$

$$+\frac{\beta\gamma}{2}\sum_{(j,j')}\left[\left(V^{ex2}\right)^{-1}\right]_{j,j'}\left(q^{INT}_j - \frac{\rho}{2}\right)\left(q^{INT}_{j'} - \frac{\rho}{2}\right)$$

$$+\sum_j\int\frac{1}{2\pi}d\vec{z}_j e^{-\frac{1}{2}\left(\vec{z}_j\right)^2}\ln\left(I_0\left(\beta\sqrt{\left(\vec{m}_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j\right)^2}\right)\right)$$

Solve $\frac{\partial F}{\partial \vec{m}_j} = 0$ and $\frac{\partial F}{\partial q^{INT}_j} = 0$!

Solve the following equations:

$$\sum_{j'\neq j}\left[\left(V^{ex} - V^{inh}\right)^{-1}\right]_{j,j'}\vec{m}_{j'} = \int\frac{1}{2\pi}d\vec{z}_j e^{-\frac{1}{2}\left(\vec{z}_j\right)^2}\frac{I_1\left(\beta\sqrt{\left(\vec{m}_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j\right)^2}\right)}{I_0\left(\beta\sqrt{\left(\vec{m}_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j\right)^2}\right)}\frac{\vec{m}_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j}{\sqrt{\left(\vec{m}_j + \sqrt{\beta^{-1}\gamma q^{INT}_j}\vec{z}_j\right)^2}}$$

$$q^{INT}_j = \frac{\rho}{2}\int\frac{d\vec{z}}{2\pi}e^{-\frac{\vec{z}^2}{2}}\left[\frac{I_1\left(\beta\left|\vec{m}_j + \sqrt{\frac{\gamma}{\beta}q^{INT}_j}\vec{z}\right|\right)}{I_0\left(\beta\left|\vec{m}_j + \sqrt{\frac{\gamma}{\beta}q^{INT}_j}\vec{z}\right|\right)}\right]^2$$

相の境界

（1）高温側で、マップ相 $m \neq 0$ と $m = 0$ ランダム相の転移温度： $T_c = \frac{\tilde{V}(k_0)}{2}$

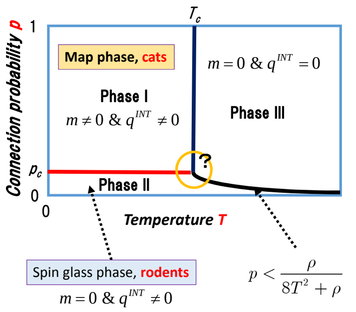
（2）マップ相では常に $q^{INT} \neq 0$

（3）高温側のランダム相で $q^{INT} \neq 0$ と $q^{INT} = 0$ の境界の曲線： $p = \frac{\rho}{8T^2 + \rho}$

（4）低温側で、マップ相とランダム相の境界： $p = p_c \equiv \frac{2\rho}{\pi\tilde{V}(k_0)^2 + 2\rho}$

$$\rho = \tilde{U}(0),\quad \tilde{U}\left(\vec{k}\right) = \int d\vec{x}\,U\left(\left|\vec{x}\right|\right)e^{i\vec{k}\cdot\vec{x}},\quad U\left(\left|\vec{x}_{i,i'}\right|\right) = U_{i,i'} = \left(V^{ex}_{i,i'}\right)^2$$

Phase diagram in OS domains

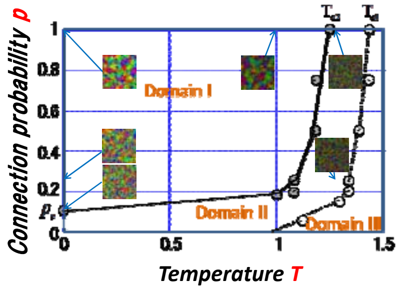


Phase I: Map representation

Phase II: Spin glass representation

Phase III: Unstable random representation

T-p space



Domain I
- Oriented RFs
- Map representation

Domain II
- Oriented RFs
- Random representation

Domain III
- Unoriented RFs

Discrepancy

Analytical estimation	Realistic simulation
$T_c \sim 0.3$ $p_c \sim 0.04$	$T_c \sim 1.2$ $p_c \sim 0.1$
↑	↑
All neurons are *a priori* orientation selective	Receptive fields of any neurons are self-organized

方位マップが種依存である要因

一次視覚野内興奮性結合のスパースネス

ネコ・フェレット：個々のシナプス結合は弱いが密

げっ歯類：　　個々のシナプス結合は強いが疎

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